Engineering Notes

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Theoretical Method for the Analysis of Airfoils in Viscous Flows

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Nomenclature

$A_{k,j}$	= influence coefficient matrix elements
b_k	= right-hand side of Eq. (10) (function of σ)
C_p	= pressure coefficients
C	= airfoil contour
f	= jump of function F across the airfoil contour
F	= function entering the Cauchy type integral, Eq. (1)
m	= number of points on the airfoil surface
u	= tangential component of velocity
v	= normal component of velocity
(x,y)	= coordinate of airfoil
z	= complex position vector
α	= angle of attack
γ	= vortex strength
σ	= source density
φ	= angle between exterior normal to C and the real axis
θ	= trailing-edge angle

Subscripts

j,k	= panel indices
L	= lower surface
U	= upper surface

Introduction

VARIOUS methods exist for the analysis of airfoils in viscous flows.[1-3] In this Note we describe yet another method for analysis of airfoils in viscous flows based upon a new vortex panel method in the complex plane for inviscid flow analysis, as outlined by Mokry.[4]

The inviscid potential flow pressure distribution about an airfloil is determined by using flat panels with linear vortex and source densities in a complex plane, the latter being used to simulate the displacement effects of the boundary layer. The boundary-layer development over the airfoil is determined by the integral technique.

Once the boundary-layer development is known, a distribution of normal component of velocity is determined as a function of the calculated boundary-layer displacement thickness and pressure distribution. The computed distribution of the normal component of velocity is included in the second calculation of the potential flow about the airfoil, and the cycle repeats until convergence is achieved. Lift, drag, and pitching moments can be determined as functions of Reynolds number.

The present method has two distinct advantages over most of the other methods currently in use. The first is that the pressure coefficients are computed directly at the contour input points, whereas most panel methods give output at panel midpoints which in fact are not points of the contour. The second is that the present method, which uses Preston's concept of finding an additional source distribution to simulate the effect of the displacement thickness of the boundary layer, preserves the airfoil geometry in the course of potential flow/boundary-layer iteration.

Method of Analysis

Inviscid Method

Let $z=x+iy$ be the complex position vector in the airfoil plane. Then consider the Cauchy type of integral

$$F(z) = \frac{1}{2\pi i} \int_C \frac{f(z')}{z'-z} dz' \tag{1}$$

where z' is a point on the contour C and f determines the jump of the function F across C, i.e., the difference of the limiting values of F_C^+ and F_C^- as z approaches the point z_0 on the contour C

$$F_C^+(z_0) - F_C^-(z_0) = f(z_0) \tag{2}$$

Now if we identify the function F with the complex disturbance velocity, then by writing $f(z')$ as

$$f(z') = -[\sigma(z') + i\gamma(z')]e^{-i\varphi(z')} \tag{3}$$

where φ is the angle between the exterior normal to C and the real axis (Fig. 1), it can be shown that the normal and tangential components of velocities are given by

$$v(z) = Re[e^{-i\alpha} + F^-(z)]e^{i\varphi(z)} = \sigma(z)$$
$$u(z) = -Im[e^{-i\alpha} + F^-(z)]e^{i\varphi(z)} = \gamma(z) \tag{4}$$

respectively, where α is the angle of attack with unit freestream velocity. The real functions σ and γ are called source and vortex densities, respectively.

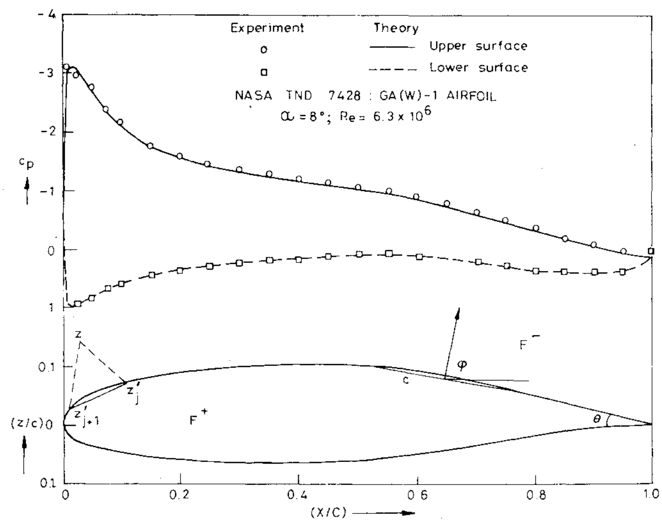

Fig. 1 Pressure distribution for NASA GA(W)-1 airfoil.

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Index categories: Subsonic and Transonic Flow; Aerodynamics.

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Here the normal component of velocity v is a given function of C. If flow is tangent to the contour, then $v(z)=0$. Nonzero values of v can be used to simulate the displacement effect of the boundary layer in terms of flow through the airfoil contour.

The trailing-edge Kutta condition of finite trailing-edge velocity becomes[4]

$$\sigma_L + i\gamma_L = -(\sigma_U + i\gamma_U)e^{i\theta} \tag{5}$$

where

$$\theta = \varphi_L - \varphi_U - \pi \tag{6}$$

is the trailing-edge angle.

Now using Eq. (1), the complex disturbance velocity at the flowfield point z can be written as

$$F^-(z) = \sum_{j=1}^{m-1} \Delta_j F^-(z) \tag{7}$$

where

$$\Delta_j F^-(z) = \frac{1}{2\pi i}\int_{z_j}^{z_{j+1}} \frac{f(z')}{z'-z}\,dz' \tag{8}$$

is the contribution of the jth panel and m is the number of corner points. Assuming f to be linear in z, Eq. (7) can be written as, along with Kutta condition[5]

$$\Lambda_{k,j}\gamma_j = b_k \tag{9}$$

$$k=1,2,\ldots,m+1 \qquad j=1,2,\ldots,m$$

which is an overdetermined system of equations for γ_j. Note that γ_j are the values of γ at the corner points and not at the panel midpoints. This overdetermined system of equations for γ_j can be solved by least squares solution.

Once the values of γ_j are known, by the Bernoulli theorem the pressure coefficients are

$$C_P(z_j) = 1 - (\sigma_j^2 + \gamma_j^2) \tag{10}$$

where σ is a given quantity.

Boundary-Layer Solution

The boundary-layer calculations are divided into three categories: laminar boundary layer, transition region, and turbulent boundary-layer calculations.

Cohen and Reshotko's method is used for the solution of the laminar boundary-layer equations. The method results from the application of Stewartson's transformation to Prandtl's equation, which yields a nonlinear set of two first-order differential equations. The equations are then expressed in terms of dimensionless parameters related to wall shear, surface heat transfer, and transformed freestream velocity. The Thwaites concept of the unique interdependence of these quantities is assumed. The evaluation of these quantities and the details of the method are outlined in detail in Ref. 5.

The Schlichting-Ulrich-Granville[6] method is used for the prediction of transition from laminar to turbulent flow.

Goradia's[1] desensitized turbulent boundary-layer equations are programmed for the turbulent boundary-layer calculations.

Combined Solution

The calculation sequence is as follows:

1) The potential flow pressure field is computed first by the method described in the inviscid flow analysis section, assuming the flow to be tangent to the airfoil contour ($v=0$).

2) The boundary-layer properties are computed as functions of the potential flow pressure distribution.

3) Based upon the concept of displacement flux by Preston, the normal component of velocity is set equal to the streamwise derivative of the product of the local potential flow velocity and the displacement thickness of the boundary layer.

4) A new potential flow solution is then computed taking into account the normal component of velocity $v(z)$ computed in step 3.

Steps 2-4 are repeated until convergence is achieved. Lift, drag, and pitching moments can thus be determined for the given airfoil in a viscous incompressible flow.

Example, Results, and Discussions

To illustrate the proposed method, the pressure distribution over the NASA GA(W)-1 airfoil was computed at various angles of attack at a Reynolds number of 6.3×10^6 for which experimental values were available.[7] The results are summarized in Figs. 1 and 2 wherein comparisons are made between the proposed theory and experimentally measured values, showing excellent agreement between the two.

The method proposed here has a number of features and capabilities. Some of these are:

1) The pressure coefficients are evaluated directly at the contour input points, whereas most panel methods give output at panel midpoints which are not points on the contour.

2) The method of representing the displacement effects of the boundary layer on the potential flow, while not necessarily more accurate than direct employment of displacement thickness, has two advantages: first, reduction of computational cost—the influence coefficient matrix need be inverted only once, with succeeding iterations requiring only matrix multiplication; second, compared to the classical approach by Prandtl, where the displacement thickness is added to the airfoil thickness to produce a modified shape, the Preston concept has the advantage of preserving airfoil shape in the course of potential flow/boundary-layer interaction.

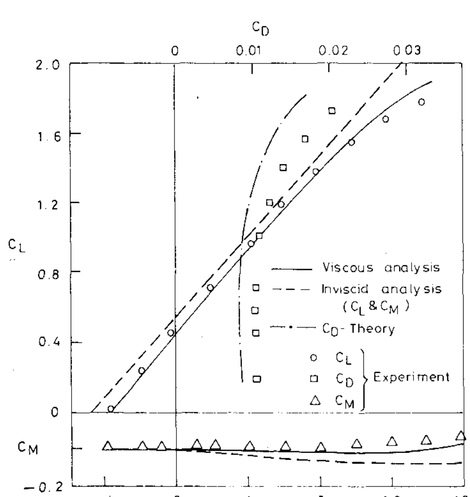

Fig. 2 Lift drag and moment coefficients for NASA GA(W)-1 airfoil.

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Model for Unsteadiness in Lateral Dynamics for Use in Parameter Estimation

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Nomenclature

b = wing span, ft
$\bar{c}$ = mean geometric chord, ft
C_l = rolling moment coefficient, $M_x/\bar{q}S_w b_w$
C_{l_β} = $\partial C_l/\partial\beta$
C_{l_p} = $\partial C_l/\partial(pb/2V)$
C_{l_r} = $\partial C_l/\partial(rb/2V)$
$C_{l_{\delta_a}}$ = $\partial C_l/\partial\delta_a$
C_n = yawing moment coefficient, $M_z/\bar{q}S_w b_w$
C_{n_β} = $\partial C_n/\partial\beta$
C_{n_p} = $\partial C_n/\partial p(b/2V)$
C_{n_r} = $\partial C_n/\partial(rb/2V)$
$C_{n_{\delta_a}}$ = $\partial C_n/\partial\delta_a$
C_y = sideforce coefficient, sideforce $/\bar{q}S_w$
C_{y_β} = $\partial C_y/\partial\beta$
C_{y_p} = $\partial C_y/\partial(pb/2V)$
C_{y_r} = $\partial C_y/\partial(rb/2V)$
g = acceleration due to gravity, ft/s^2
I_x = roll moment of inertia, slug-ft^2
I_z = yaw moment of inertia, slug-ft^2
l_v, Z_v = coordinates of the quarter-chord point of the mean chord of the vertical tail with respect to the center of gravity of the airplane, ft
m = mass, slugs
M_x, M_z = moment about the roll and yaw axes, respectively, lb-ft
p = rate of roll, rad/s
$\bar{q}$ = dynamic pressure, lb/ft^2
r = rate of yaw, rad/s
S = wing surface area, ft^2
t = time, s
V = freestream velocity, ft/s
x = state vector
β = sideslip angle, rad
σ = sidewash angle, rad
ρ = air density, slugs/ft^3
ω = angular frequency, rad/s
ϕ = roll angle, rad
δ_a = aileron deflection, rad
$\Delta\sigma(t)$ = indicial sidewash function
$\Delta C_y(t)$ = indicial sideforce function

Subscripts

F = fuselage
ss = steady state (no unsteady aerodynamic effects)
v = vertical tail
w = wing

Superscript

$(\tilde{\ })$ = Fourier transform of the variable in parentheses

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Introduction

DURING the past two years, some interest and research effort has been given to the role of unsteady aerodynamics in aircraft dynamics. In particular, it was shown that the effects of unsteadiness should be included in a parameter estimation algorithm for longitudinal stability and control derivatives for additional confidence in the estimates. Since existing methods of computing the unsteady lateral derivatives are so limited, this research program was begun.

Analysis

Unsteadiness is introduced into the lateral dynamics via the indicial sideforce and sidewash angle produced by a unit step change in sideslip angle. Due to the changes in magnitude and direction of the flow at the vertical tail from the wing, fuselage and possible propeller slipstream, the effective angle of attack of the vertical tail differs from the sideslip angle by the sidewash angle σ. The sidewash in sideslipping flight is analogous to the downwash in the longitudinal case. Consequently, the change in sidewash angle due to a unit step change in sideslip is assumed to vary as the shed wake moves along the aircraft according to[1]:

$$\Delta\sigma_{l_v}(t) = \left(\frac{\partial\sigma}{\partial\beta}\right)_{ss,l_v}\left\{I - \frac{F}{\left[\dfrac{l_v\cos(I)}{\bar{c}_w} - I - \dfrac{Vt}{2\bar{c}_w}\right]} - G\exp\left(\frac{-HVt}{\bar{c}_w}\right)\right\} \tag{1}$$

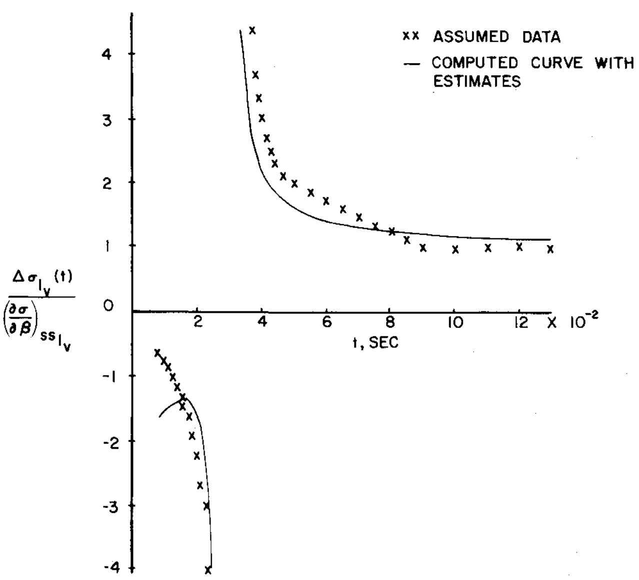

Fig. 1 Comparison of assumed and computed sidewash at the vertical tail.